

Internal structural optimization of hollow fan blade based on sequential quadratic programming algorithm

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Abstract: Several structural design parameters for the description of the geometric features of a hollow fan blade were determined. A structural design optimization model of a hollow fan blade which based on the strength constraint and minimum mass was established based on the finite element method through these parameters. Then, the sequential quadratic programming algorithm was employed to search the optimal solutions. Several groups of value for initial design variables were chosen, for the purpose of not only finding much more local optimal results but also analyzing which discipline that the variables according to could be benefit for the convergence and robustness. Response surface method and Monte Carlo simulations were used to analyze whether the objective function and constraint function are sensitive to the variation of variables or not. Then the robust results could be found among a group of different local optimal solutions.

Key words: hollow fan blade; structural optimization; sequential quadratic algorithm;
finite element method; Monte Carlo simulations

CLC number: V231.91

Document code: A

Fan blade has been one of the key structures of the turbofan engine. Currently, there are many new trends in structure design of fan blade, such as the titanium hollow fan blade with lower aspect ratio^[1-2]. The most significant feature of a hollow fan blade is that the pressure surface and suction surface are produced by titanium thin sheet which envelop a cavity in the blade, and there are titanium ribs looks like truss in the cavity. And super-plastic forming/diffusion bonding process is widely used in the manufacture of hollow fan blades^[3-4].

The design of a hollow fan blade is a typical structural optimization problem because it is essential to design the proper form, size and internal ribs of the cavity^[5].

Structural optimization design usually consists of three basic types^[6]: topology optimization, shape optimization and size optimization. Topology optimization^[7-8] which is often used in

planar structure is often the first step in structural optimization design, but it is difficult to obtain satisfactory results when using this method for three-dimensional solid structures because of its inherent drawback. Second-generation wide chord hollow fan blade of Rolls-Royce^[5] has simple structure of the internal cavity, and it has shown good performance in structural integrity and durability in the process of testing and real application. The cross section topology sketch of this blade can be seen in Fig. 1. Therefore, this typical topology will be used in the structure shape and size optimization of a hollow fan blade based on finite element method in this paper.

There are many mathematical methods for structural optimization, including mathematical programming algorithms and heuristic intelligent optimization algorithms. Although heuristic optimization algorithms generally have a tendency

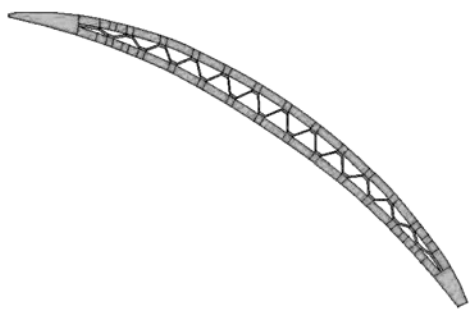


Fig. 1 Typical cross section of a hollow fan blade model

to find the global optimal result, the time costs are usually so enormous that it could not be accepted for real engineering structural optimization based on finite element analysis^[9-10]. Therefore, the sequential quadratic optimization algorithm was chosen for its robustness and efficiency^[11-12]. Although this algorithm does not guarantee convergence to the global optimum, it can significantly improve the efficiency of searching the local optimal solutions with the greatly reduction of analysis time. Because of the characteristic of the sequential quadratic algorithm, the optimal results are sensitive to the initial value of design variables. To overcome this drawback, a group of different initial values should be determined to obtain a better result. When the local optimal solutions are obtained, it is very important to evaluate which one is better than the others. However, in practical structures, the better solution means not only the smaller objective but also the smaller dispersion of constraint and objective value because of the variables fluctuation. Therefore, the response surface method and Mento Carlo simulation^[13-14] are used to analyze the effects of design variables on the objective and constraint functions, for the purpose of finding better and more robust solution.

1 Optimization model of a hollow fan blade

1.1 Design variables

Four basic structure design parameters of a hollow fan blade were determined in this paper,

the structure, form and size of a hollow fan blade and the inner cavity can be uniquely established based on these parameters.

The first, thickness of the thin sheet which was used to form the pressure surface and the suction surface is signed by letter D shown in Fig. 2. It should be noted that the thickness are different from the root to the top of a blade, so the thickness of the thin sheet for eight different cross sections which are equidistant distribution from the root of cavity to the top are selected, and then the thickness of thin sheet in any cross section could be interpolated by this group of thickness. So the shape and size of cavity are determined uniquely.

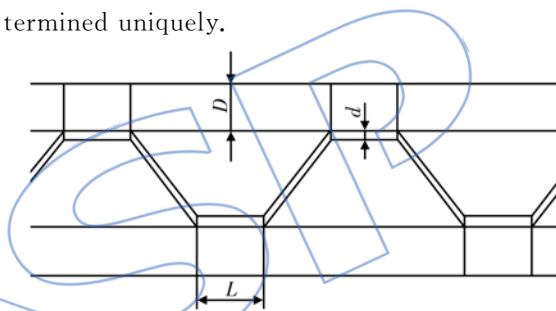


Fig. 2 Local sketch for cross section of a hollow fan blade model

The second parameter is the thickness ratio between the rib in cavity and the thin sheet. The thickness of rib which is signed by letter d in Fig. 2 can be uniquely determined by the thickness of thin sheet and the thickness ratio.

The third, diffusion bonding ratio. It is the ratio between the length of all diffusion bonding regions and the average arc length of both sides of the cavity in a cross section. Once the shape and size of cavity is determined by the thickness of thin sheet, the length of the bonding regions which is signed by letter L in Fig. 2 could be uniquely determined by the bonding ratio.

The fourth, number of the ribs in the cavity. A cross section of a hollow fan blade with 24 ribs is shown in Fig. 1.

1.2 Structural optimization model of a hollow fan blade

Once the structural optimization design parameters are determined, the finite element(FE)

model of a hollow fan blade shown in Fig. 3 and Fig. 4 can be established. Then, the maximum Mises equivalent stress of the hollow fan blade is chosen as constraint, and the objective is the minimum mass. So the optimization design of the hollow fan blade can be described in formula:

$$\begin{aligned} \min \quad & \text{mass}(\mathbf{X}) \\ \text{s. t.} \quad & \text{SeqvMax}(\mathbf{X}) \leq \sigma_{\max} \\ & \mathbf{X}_l \leq \mathbf{X} \leq \mathbf{X}_u \end{aligned}$$

Where, $\text{mass}(\mathbf{X})$ means the mass of the hollow fan blade, $\text{SeqvMax}(\mathbf{X})$ means the maximum Mises equivalent stress, $\sigma_{\max}$ means the constraint of the $\text{SeqvMax}(\mathbf{X})$, and the vector $\mathbf{X} = (D_1, D_2, D_3, D_4, D_5, D_6, D_7, D_8, C, L, N)^T$ means the design variables. In the expression of vector $\mathbf{X}$, D_i means the thickness of thin sheet of the number i cross section among the eight different cross sections mentioned above, C means the thickness ratio, L means the diffusion bonding ratio, and N means the number of ribs. The capacity of the hollow fan blade bearing with surface pressure and bending load will be improved for the increase of number of ribs in cavity. But the more there are the ribs, the shorter the length of diffusion bonding region will be. Therefore, the number of ribs should be chosen as more as possible on the basis of ensuring bonding strength. So 24 is adopted as the number of ribs for the hollow fan blade used in this paper, and then the parameter N mentioned in formula is no longer variable but constant.

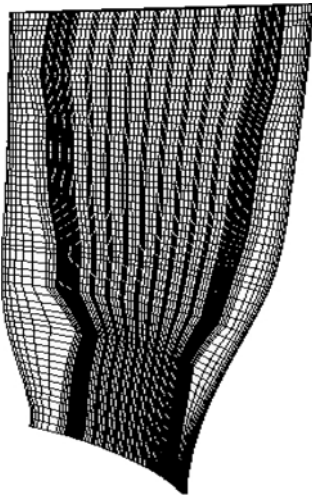


Fig. 3 FE mesh of a hollow fan blade

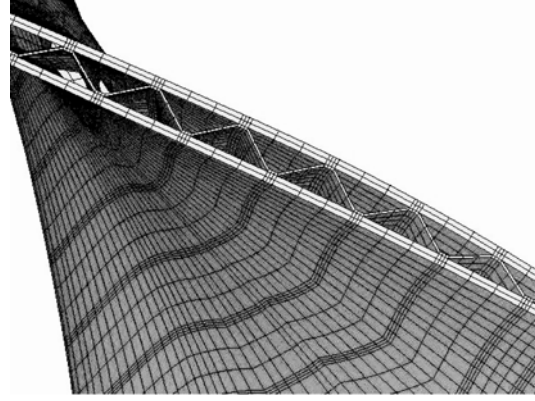


Fig. 4 Local sketch for FE mesh of a hollow fan blade

2 Sequential quadratic programming algorithm

Sequential quadratic programming algorithm which is efficient and robust is commonly used in the nonlinear optimization problem with constraints. The objective and constraint functions of original optimization problem are gradually approached through the successive quadratic functions. A typical non-linear optimization problem with constraints can be described as:

$$\begin{cases} \min & f(\mathbf{x}) \\ \text{s. t.} & c_i(\mathbf{x}) = 0, \quad i \in E = \{1, 2, \dots, m\} \\ & c_j(\mathbf{x}) \leq 0, \quad j \in I = \{m+1, m+2, \dots, p\} \end{cases}$$

Where the $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$ means the design variables vector, $f(\mathbf{x})$ means objective function, and $c_i(\mathbf{x})$, $c_j(\mathbf{x})$ means equal and unequal constraint functions. The basic idea of sequential quadratic programming algorithm is that the objective function and constraint functions can be approximated by the Taylor expansion in the k -th approximation solution. Suppose that $\mathbf{x}^k$ is the k -th approximation solution, as a result, the original problem can be approximated by^[12]:

$$\begin{cases} \min & Q(\mathbf{d}) = \frac{1}{2} \mathbf{d}^T \mathbf{B}_k \mathbf{d} + \nabla f(\mathbf{x}^k)^T \mathbf{d} \\ \text{s. t.} & \nabla c_i(\mathbf{x}^k)^T \mathbf{d} + c_i(\mathbf{x}^k) = 0, \quad i \in E \\ & \nabla c_j(\mathbf{x}^k)^T \mathbf{d} + c_j(\mathbf{x}^k) \leq 0, \quad j \in I \end{cases}$$

Where $Q(\mathbf{d}) = f(\mathbf{x}^{k+1}) - f(\mathbf{x}^k)$, $\mathbf{B}_k = \nabla^2 f(\mathbf{x}^k)$.

The matrix $\mathbf{B}_k$ is usually approximated using the BFGS (Broyden-Fletcher-Goldfarb-Shanno) formula, because exact calculations require information concerning the second derivative and second derivatives are computationally extremely expensive to obtain. $\nabla f(\mathbf{x}^k)$ and $\nabla c_i(\mathbf{x}^k)$ are usually calculated using forward-differences. Then the original non-linear optimization problem with constraints is converted to a quadratic programming problem and the search direction vector $\mathbf{d}$ can be determined by solving the formula. Then, the design variables are updated using $\mathbf{x}^{k+1} = \mathbf{x}^k + \alpha \mathbf{d}$ where α is obtained by performing a line search in the range $\alpha \in [0, 1]$. Finally, this process is repeated until convergence.

3 Optimization process of a hollow fan blade

In this section, the structure of the titanium hollow fan blade is optimized using the sequential quadratic programming algorithm.

3.1 Finite element analysis parameters

Material properties: the density is $4.44 \times 10^3 \text{ kg/m}^3$, Young's modulus is 109 GPa, the Poisson's ratio is 0.2386.

Boundary conditions: the rigid body displacement is zero in the root of the fan blade.

Load conditions: the rotor speed is 1 000 rad/s, pressure on pressure surface is 0.15 MPa, the pressure on suction surface is 0.1 MPa.

3.2 Finite element analysis of a solid fan blade

It is important to determine what maximum Mises equivalent stress should be given as the constraint. So at first, a solid fan blade as the same shape, size and load conditions as the hollow fan blade is analyzed using finite element method for the purpose of finding the maximum Mises equivalent stress which will be used as the constraint for the optimization. The Mises equivalent stress distribution contour of the solid blade is shown in Fig. 5.

The mass of the solid blade is 0.85 kg, and the maximum Mises equivalent stress which appears on the suction surface is 465.5 MPa. The

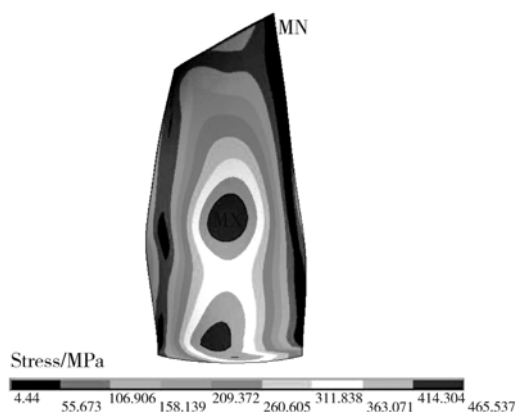


Fig. 5 Mises equivalent stress distribution of the solid blade

value of maximum Mises equivalent stress is lower on the pressure surface than on the suction surface, but it is higher than 400 MPa still, and its location is lower than the position where the maximum Mises equivalent stress of suction surface appears. So the location of the cavity root in the hollow fan blade should be laid on the position between these two regions, for the purpose of staying away from the root of the solid blade and those regions with large stress.

3.3 Structural optimization of the hollow fan blade

The purpose of the optimization design is finding an acceptable structure of the hollow fan blade with minimum mass under the constraint that the maximum Mises equivalent stress is not larger than 465 MPa. The value ranges of the design variables are listed in Table 1.

Several primary aspects are taken into account for the choice of the value ranges. First, the hollow fan blade based on the value of design variables will be not failure or buckling under the load conditions. Second, the requirements of the diffusion bonding strength should be met, this aspect is limited by the technology of manufacture. Third, the geometrical coordination should be satisfied, that means a real structure of a hollow fan blade could be established through the combination of these values.

There are five groups of initial value of design variables in Table 2. These different initial design variables would be result in different local

Table 1 Value ranges of the design variables for the hollow fan blade

	D_1/mm	D_2/mm	D_3/mm	D_4/mm	D_5/mm	D_6/mm	D_7/mm	D_8/mm	C	L
Upper limit	1.2	1.2	1.0	1.0	1.0	1.0	0.8	0.8	0.2	0.2
Lower limit	2.2	2.0	1.8	1.8	1.6	1.6	1.6	1.0	0.4	0.6

Table 2 Initial values of design variables for structural optimization of the hollow fan blade

Groups	D_1/mm	D_2/mm	D_3/mm	D_4/mm	D_5/mm	D_6/mm	D_7/mm	D_8/mm	C	L
1	1.5	1.5	1.5	1.5	1.5	1.5	1.5	0.8	0.3	0.4
2	2.2	2.0	1.8	1.6	1.4	1.2	1.0	0.8	0.3	0.4
3	1.74	1.51	1.31	1.28	1.27	1.26	1.2	1.0	0.3	0.4
4	2.2	1.8	1.4	1.0	1.0	1.0	0.8	0.8	0.25	0.2
5	1.7	1.4	1.2	1.0	1.0	1.0	0.8	0.8	0.2	0.2

optimal solutions.

These five groups initial value of design variables are chosen for the purpose of not only finding better solutions but also exploring which kind of discipline that the design variables are according to could lead to better results. For the first group of initial value, the thicknesses of thin sheets are equal except for D_8 . For the second group, the thickness of thin sheet changes in accordance with the linear discipline from the root to the top in the cavity. For the third group, the thickness of thin sheet changes in accordance with the same discipline which the thickness of solid blade distributes according to. For the fourth and the fifth groups, the D_1 , D_2 and D_3 are changes according to linear discipline,

but the values are different. The value of parameters C and L are chosen without any special rules, they are only litter different among these five groups. Then, five groups of corresponding optimal solutions which are shown in the first five columns of Table 3 are obtained using sequential quadratic algorithm.

It should be noted that the solution shown in the first column of Table 3 is not well, because it is heavier than the other solutions, and the D_3 is much bigger than D_2 , this phenomenon should not be accepted in a real hollow fan blade. But if the solving procedure of the fist optimal solution is observe carefully, a better local solution will be found by releasing the constraint of maximum Mises stress to 467 MPa. This solution

Table 3 Optimal solutions for structural optimization of the hollow fan blade

Variables	1	2	3	4	5	6
D_1/mm	2.196	1.817	2.2	2.00	1.751	1.905
D_2/mm	1.338	1.350	1.327	1.316	1.364	1.310
D_3/mm	1.771	1.224	1.236	1.382	1.315	1.291
D_4/mm	1.0	1.071	1.0	1.0	1.0	1.007
D_5/mm	1.0	1.021	1.0	1.0	1.0	1.0
D_6/mm	1.0	1.0	1.0	1.0	1.0	1.0
D_7/mm	0.8	0.8	0.8	0.8	0.8	0.8
D_8/mm	0.8	0.8	0.8	0.8	0.8	0.8
C	0.36	0.2	0.2	0.2	0.27	0.2
L	0.2	0.38	0.60	0.2	0.21	0.238
Max Mises stress/MPa	463.92	464.5	465.00	464.96	464.99	466.96
Mass	0.6913	0.6537	0.6566	0.6574	0.6614	0.6529
Iteration numbers	180	185	211	158	113	229

with released constraint is shown in the sixth column of Table 3. It can be seen that the mass reduces sharply. The reason of this phenomenon is that there are multi-peaks for the objective function and constraint function, so the constraint condition should be met first in the process of optimization, and then the optimal solution are solved in the feasible region.

The optimal solutions of D_4 to D_8 are very similar although the initial values are very different in Table 3. Especially for the first three columns and the sixth column, although the D_4 to D_8 are variables in the procedure of optimization, the optimal solutions are almost equal to the corresponding solutions in the fourth and the fifth columns. This shows that the lower limit of initial values could be chosen for these variables corresponding. It seems that if some variables could be removed before the optimization process, the efficiency can be improved significantly.

In the manufacture procedure of a real hollow fan blade, it is inevitable that the values of all of these design variables will be scatter in a region around the optimal values with a standard deviation. It will definitely lead to a fluctuation of maximum Mises equivalent stress and mass of the hollow fan blade. Therefore, the best design is not only that the optimal solution will meet the requirements of structural performance with minimum mass but also it will not be excessively sensitive to the small change of design variables. Because of this reason, for the purpose of evaluating the optimal solutions in Fig. 3 comprehensively, it is assumed that every design variable scatters around the corresponding optimal solution with a fluctuation no more than $\pm 5\%$. So

every design variable fits the normal distribution $N(\mu_i, \sigma_i)$, where μ_i represents the mean value of the design variable i , it equals to the value of corresponding optimal solution, and $\sigma_i = \frac{1}{3} \times (0.05\mu_i)$ represents the standard deviation of the design variable i . Then a local quadratic polynomial response surface model which fit the relationship between the mass or the maximum Mises stress and the design variables are established using central composite design sampling method. 100 000 times Monte Carlo simulations are employed to obtain the distribution of the mass and the maximum Mises equivalent stress, the mean value and standard deviation are shown in Table 4. It should be noted that the first group of optimal solutions in Table 3 are not simulated through Monte Carlo method, because the solution is not well, but the release solution shown in the sixth column of Table 3 is used instead.

In Table 4, μ_{mass} and μ_{stress} represent the mean value of the mass and the maximum Mises equivalent stress respectively, and σ_{mass} and σ_{stress} represent the standard deviation of the mass and the maximum Mises equivalent stress. The number of groups in Table 4 is corresponding to Table 3.

Although it can be seen that the best results may be group 2 or group 6 in Table 3, for their mass are lighter than the other groups, they do not show significant advantages in Table 4. It is difficult to choose the best solution in Table 4, all of the μ_{stress} exceeds the constraint, but they are similar, and the deviation appears no special rules. So, all of these solutions can be

Table 4 Results of Monte Carlo simulations for each optimal solution

Results	2	3	4	5	6
$\mu_{\text{mass}}/\text{kg}$	0.6529	0.6566	0.6573	0.6614	0.6529
$\sigma_{\text{mass}}/\text{kg}$	0.00134	0.00133	0.00135	0.00141	0.00132
$\mu_{\text{stress}}/\text{MPa}$	468	469	468	469	470.0
$\sigma_{\text{stress}}/\text{MPa}$	5.574	4.94	5.06	5.55	4.57

considered as optimal solution. But group 3 is not recommended, for the diffusion bonding ratio is too larger, that means more difficult to manufacture.

4 Conclusions

Sequential quadratic programming algorithm is used for the internal structural optimization of a hollow fan blade based on finite element method. Several useful conclusions could be obtained through the analysis of optimization process and the optimal solutions:

Several design parameters for description of the geometric features of a hollow fan blade are determined. These parameters should meet the requirement of geometric coordination, and the hollow fan blade could be established uniquely and manufactured.

In the process of structural design optimization of a hollow fan blade, the thickness of thin sheet of the pressure surface and the suction surface near the root of cavity is the most important. They should be chosen carefully. The thickness ratio ranged from 0.2 to 0.3 and diffusion bonding ratio ranged from 0.2 to 0.4 are recommended.

Sequential quadratic programming algorithm is efficient and robust. Although this method does not guarantee convergence to global optimal solution, some similar local optimal solutions could be obtained through different initial value of design variables.

It is recommended that the initial thickness values of thin sheets from the root to the top of cavity could be in accordance with the linear discipline. The optimization process can converge to a better local solution robustly based on this option, and the searching efficiency could be improved significantly.

The response surface method and Monte Carlo simulation could be used to analyze whether the objectives and the constraints are sensitive to the variation of design variables or not, then a better optimal solution could be selected by the

comprehensive comparison among several results.

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